Printed Pages : 4

ECS-303

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 110303

Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) (ODD SEM.) THEORY EXAMINATION, 2014-15

DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours] [Total Marks : 100

Note : (1) Attempt all questions.

(2) Make suitable assumptions wherever necessary.

1 Attempt any **four** parts of the following : **(5×4=20)**

(a) Let A be a set with 10 distinct elements. Determine the following :

(i) Number of different binary relations on A.

(ii) Number of different symmetric binary relations on A.

(b) How many different reflexive, symmetric relations are there on a set with three elements ?

(c) Prove that union of two countably infinite set is countably infinite.

(d) Composition of functions is commutative. Prove the statement or give counter example.

(e) Show that $102^{n+1}+1$ is divisible by 11, $\forall n \in N$.

(f) What do you understand by asymptotic behavior of a numeric function ? Explain. Big-oh and Big-Omega notation.

2 Attempt any **two** parts of the following : **(10×2=20)**

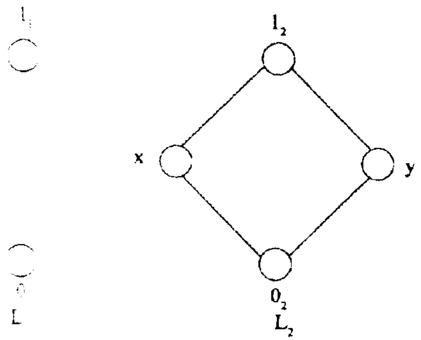
(a) Show that (R-{1}, *) where the operation is defined as a*b = a + b − ab is an abelian group.

(b) Discuss all properties of subgroups and Lagrange's theorem.

(c) Show that if $f: G \rightarrow G'$ is an isomorphism and G is an abelian group, then G' is also abelian.

3 Attempt any **two** parts of the following : **(10×2=20)**

(a) Let $(L_1, \leq)$ and $(L_2, \leq)$ be lattices as shown below. Then draw the Hasse diagram for the lattice $(L, \leq)$, where $L = L_1 \times L_2$.



(b) Describe the Boolean duality principle. Write the dual of each Boolean equations:

(i) $x + \bar{x}y = x + y$

(ii) $(x.1) + (\bar{x} + 0) = 0$

(c) Define a Poset. Show that "less than or equal to" relation on set of real number is partial Ordering. Draw the Hasse diagram of the following set under the partial ordering relation "Divides": {2, 4, 8, 16}.

4 Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) Show that the statement

$P \rightarrow Q$ *and* $\neg Q \rightarrow \neg P$ are equivalent.

(ii) Write the negation and contrapositive of the following statements:

a. If he walks, he will be healthy.

b. Only if mohan works hard, he will pass the test.

(b) Convert the following into CNF.

(i) $p \wedge (p \rightarrow q)$

(ii) $\neg(p \vee q) \leftrightarrow (p \wedge q)$

(c) (i) Show that the premises "everyone in this Discrete Mathematics class has taken a course in CS" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in CS".

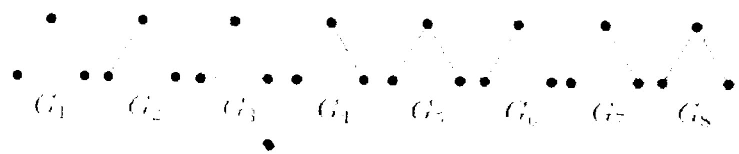
(ii) Find the negation of the following statements:

a. $(\forall x.P(x)) \vee (\exists y.P(y))$

b. $\forall x \in R, x > 5 \rightarrow x^3 > 125$

5 Attempt any **two** parts of the following: **(10×2=20)**

(a) There are eight graphs on three vertices, and they are shown below:



(i) Determine the number of graphs on n vertices.

(ii) Determine the number of graphs on n vertices with exactly m edges.

(b) Suppose that a connected graph G has 11 vertices and 53 edges

(i) Show that G is not Eulerian.

(ii) Show that G is Hamiltionian.

(c) Solve the recurrence relation using the method of generating functions :

$$a_{n+2} - 5a_{n+1} + 6a_n = 2,\quad n \geq 0,\quad a_0 = 3,\quad {}_1 = 7.$$
